

58-30 30
232-100
108
N90-19946
WG 032 961

EXPERIMENTAL CONSTRAINTS ON METRIC AND NON-METRIC THEORIES OF GRAVITY

CLIFFORD M. WILL

*McDonnell Center for the Space Sciences, Department of Physics
Washington University, St. Louis, Missouri 63130*

ABSTRACT

Experimental constraints on metric and non-metric theories of gravitation are reviewed. Tests of the Einstein Equivalence Principle indicate that only metric theories of gravity are likely to be viable. Solar-system experiments constrain the parameters of the weak-field, post-Newtonian limit to be close to the values predicted by general relativity. Future space experiments will provide further constraints on post-Newtonian gravity.

I. INTRODUCTION

Gravitation plays a fundamental role in our universe. On a local scale, up to 10^9 km, it determines our Earthbound environment, the nature of the Sun, the dynamics of the solar system. On scales ranging up to the largest observable distances, 10^{10} light years, it determines the structure and evolution of black holes, galaxies, clusters and superclusters of galaxies, and the universe itself. On scales ranging down to the smallest, the Planck scale, or 10^{-33} cm, gravitation forms the template against which must be meshed attempts to unify the interactions in a full quantum synthesis. It is remarkable that there exists one candidate theory of gravity, general relativity, that has the ability to treat gravitation over such a range — 60 orders of magnitude — of scales.

On the other hand, the viability of general relativity is determined by experiments that, with a few exceptions, are confined to the scale of the solar system. During the past 25 years, experiments have been spectacularly successful in verifying general relativity over this scale, and in ruling out many alternative theories of gravity. Space experiments, involving spacecraft tracking, orbiting atomic clocks, laser ranging to retroreflectors, and the like, have played a vital role in this endeavor.

But the need to extrapolate gravitational theory from solar system scales to such large and such small scales requires the most accurate verification possible at the experimentally accessible scales. Thus, despite its successes, experimental gravitation continues to be an active and challenging field, with space experiments maintaining their central role. In this paper we review the current status of experimental constraints on gravitational theory and describe the significance of future measurements.

II. CONSTRAINTS ON THEORIES OF GRAVITY: THE PRESENT PICTURE

One of the fundamental postulates of gravitational theory is the Einstein Equivalence Principle (EEP), which states: (i) test bodies fall with the same acceleration (weak equivalence principle — WEP); (ii) in a local freely falling frame, non-gravitational physics is independent of the frame's velocity (local Lorentz invariance); and (iii) in a local freely falling frame, non-gravitational physics is

independent of the frame's location (local position invariance). If EEP is valid, then gravity must be described by a "metric theory," whose postulates are that there exists a symmetric metric $g_{\mu\nu}$, whose geodesics are the trajectories of structureless test bodies and, which reduces to the Minkowski metric in freely falling frames, where the laws of physics take their special relativistic forms. The EEP divides theories of gravity into two classes: metric theories, such as general relativity, the Brans-Dicke theory, and numerous others; and non-metric theories, such as Moffat's non-symmetric gravitation theory (NGT), and others.

The observational evidence in support of EEP is very strong. For example, Eötvös-type experiments have verified WEP to better than a part in 10^{11} and improved space-borne experiments are planned. Local Lorentz invariance has been verified to high precision by several extraordinarily precise "mass-anisotropy" null experiments. Finally, gravitational redshift experiments test local position invariance: the 1976 rocket experiment (NASA's GP-A) verified this effect to two parts in 10^4 . It should be noted that redshift experiments that are sensitive to effects at second order in the gravitational potential probe beyond local position invariance and *do* test alternative metric theories of gravity. (For a review of the theoretical and observational implications of EEP, see Will (1981), chapter 2; or Will (1984), sec. 2; see also Haugan and Will (1987).)

The experimental evidence in support of EEP suggests very strongly that metric theories provide the best description of gravitation. When we restrict attention to such theories and consider the weak-field, slow-motion limit appropriate to the solar system, the so-called post-Newtonian limit, then it turns out that most such theories can be described by the parametrized post-Newtonian (PPN) formalism (for a detailed review, see Will (1981), chapter 4; or Will (1984), sec. 3.3). This formalism characterizes the metric of the post-Newtonian limit in terms of a set of ten dimensionless parameters, $\gamma, \beta, \xi, \alpha_1, \alpha_2, \alpha_3, \zeta_1, \zeta_2, \zeta_3, \zeta_4$, whose values vary from theory to theory. Table 1 shows the approximate significance of these parameters, and gives their values in general relativity and in theories of gravity that possess conservation laws for momentum (semi-conservative theories and all Lagrangian-based theories) and that possess conservation laws for momentum as well as angular momentum and center-of-mass motion. Several compendia of alternative theories and their PPN parameter values have been published (see for example Will (1981), chapter 5; or Will (1984), sec. 3.4). In addition to its use as a tool for studying and classifying theories of gravity, the PPN formalism facilitates discussion of experiments because the predicted sizes of various post-Newtonian effects depend on the values of the PPN parameters; therefore the measurement of an effect is tantamount to a measurement of the corresponding PPN parameter or parameter combination.

Two important experimental tests of general relativity are the deflection of light and the Shapiro time delay of light, both measuring the same thing, the coefficient $\frac{1}{2} (1 + \gamma)$. A light ray which passes the Sun at a distance d (measured in solar radii) is deflected by an angle

$$\Delta\theta = \frac{1}{2} (1 + \gamma) 1.75/d , \quad (\text{II.1})$$

Table 1. The PPN Parameters and Their Significance*

Parameter	What it measures relative to general relativity	Value in general relativity	Value in semi-conservative theories	Value in fully-conservative theories
γ	How much space-curvature is produced by unit rest mass?	1	γ	γ
β	How much "nonlinearity" is there in the superposition law for gravity?	1	β	β
ξ	Are there preferred location effects?	0	ξ	ξ
α_1	Are there preferred-frame effects?	0	α_1	0
α_2			α_2	
α_3			0	
			0	
ζ_1	Is there violation of conservation of total momentum?	0	0	0
ζ_2			0	
ζ_3			0	
ζ_4			0	

*For a compendium of PPN parameter values in alternative theories together with derivations, see TEGP, Chapter 5.

and a light ray which passes the Sun on a round trip, say, from Earth to Mars at superior conjunction, suffers a delay given, for $d \gtrsim 1$, by

$$\Delta t = \frac{1}{2}(1 + \gamma)250(1 - 0.161nd) \mu s . \quad (\text{II.2})$$

Measurements of the deflection of light have improved steadily during the past 70 years, from the early observations of stellar positions surrounding total solar eclipses (10 to 30%), to measurements of the deflection of radio waves from quasars during the period 1969 through 1975 (1.5 %), to VLBI observations of radio source positions over the entire celestial sphere in the 1980s (approaching 1%) (Will (1984), sec. 4.1; Robertson and Carter (1984).) Orbiting optical interferometers may yield further improvements, and have the potential to probe second-order, post-post-Newtonian contributions to the deflection.

Observations of the Shapiro time delay began in the middle 1960's using radar echos from Mercury and Venus, and later made use of interplanetary spacecraft

equipped with radar transponders, such as Mariners 6, 7, and 9, and the Viking landers and orbiters. Data from Viking produced the best measurement of $\frac{1}{2}(1 + \gamma)$ to date, namely 1.000 ± 0.001 , in complete agreement with general relativity (Will (1984), sec. 4.2). The time delay in a one-way signal has been recently measured using timing data from the millisecond pulsar PSR 1937+21, with results in agreement with general relativity at the three percent level (Taylor, 1987).

The perihelion shift of Mercury is another key test of general relativity. Including the possible effect of a solar quadrupole moment J_2 , the predicted rate of advance is given, in arcseconds per century, by

$$d\omega/dt = 42''98\lambda, \quad (\text{II.3})$$

$$\lambda \equiv \frac{1}{3}(2 + 2\gamma - \beta) + 0.0003(J_2/10^{-7}). \quad (\text{II.4})$$

The first term in the coefficient λ is the "classical" relativistic perihelion shift contribution, which depends on the PPN parameters γ and β . In general relativity, this term is unity (see Table 1). The second term depends on the Sun's oblateness; for a Sun that rotates uniformly with its observed surface angular velocity, so that the oblateness is produced by centrifugal flattening, J_2 is estimated to be 10^{-7} , so that in such a case, its contribution to λ would be very small.

Now, the measured shift is known accurately: after the perturbing effects of the other planets have been accounted for, the excess perihelion shift is known to be about 0.5% from radar observations of Mercury since 1966, with the result that $\lambda = 1.003 \pm 0.005$. If J_2 were indeed as small as 10^{-7} , this would be in complete agreement with general relativity. However, over the past 25 years, a range of values has been reported for J_2 , from 2.5×10^{-5} , inferred from 1966 visual solar-oblateness measurements, to a few parts in 10^6 from 1983 to 1985 visual observations, to an upper limit of 3×10^{-6} inferred from combined Mercury/Viking Mars ranging data, to $(1.7 \pm 0.4) \times 10^{-7}$ inferred from solar oscillation data (for a review, see Will (1984), sec. 4.3 and 4.4; and Will (1987), sec. 5.4.1). Thus, there remains some uncertainty in the interpretation of perihelion shift measurements as tests of general relativity, although conventional wisdom points toward the smaller values of J_2 . An unambiguous measurement of J_2 through direct study of the Sun's gravitational field over a large range of distances could be provided by a space mission that has been under study by NASA since 1978. Known as Starprobe, it is a spacecraft that would approach the Sun to within four solar radii. Feasibility studies indicate that J_2 could be measured to an accuracy of ten percent of its conventional value of 10^{-7} . Unfortunately, it is not clear whether gravitational physics is part of NASA's current plan for this mission.

Another class of experiments tests what is called the Strong Equivalence Principle (SEP). This is a stronger principle than EEP, stating that *all* bodies, including those with self-gravitational binding energy (stars, planets), should fall with the same acceleration, and that in suitable "local" freely falling frames, the laws of *gravitation* should be independent of the velocity and location of the frame. General relativity satisfies SEP, but most other metric theories of gravity do not. Lunar laser ranging measurements since 1969 have shown that the Earth and the

Moon fall toward the sun with the same acceleration to 7 parts in 10^{12} , yielding the limit (Bender, 1988)

$$|4\beta - \gamma - 3 - \frac{10}{3}\xi - \alpha_1 + \frac{2}{3}\alpha_2 - \frac{2}{3}\zeta_1 - \frac{1}{3}\zeta_2| < 0.007 . \quad (\text{II.5})$$

If the laws of gravitation in a local system (for example, the locally measured Newtonian gravitational constant) depend on the motion of the system relative to the universe, then, according to the PPN formalism, there should occur such effects as anomalous Earth tides and variations in the Earth's rotation rate, anomalous contributions to the perihelion shifts for Mercury and Earth, self-accelerations of pulsars, and anomalous torques on the Sun that would cause its spin axis to be randomly oriented relative to the ecliptic, all among other anomalies known generically as "preferred frame" effects. Negative searches for these effects have produced strong constraints on the PPN parameters α_1 , α_2 , α_3 , and ξ . A possible cosmological variation in Newton's gravitational constant has been constrained by analysis of Viking ranging data to be less than 10^{-11} yr^{-1} (for a review of tests of SEP, see Will (1984), sec. 5; and Nordvedt (1987).) Apart from indirect limits, such as that shown in equation (5), the only strong limit on the conservation-law parameters ζ_i is

$$|\zeta_3| < 10^{-8} . \quad (\text{II.6})$$

from a test of Newton's third law using the Moon (Bartlett and van Buren, 1986).

The current best limits on PPN parameters are summarized in Table 2. General relativity is consistent with all of them.

III. CONSTRAINTS PROVIDED BY PLANNED OR PROPOSED PROJECTS

There are numerous ideas for probing the structure of gravity in the solar system to higher precision. Some of them provide improved values of PPN parameters, some measure PPN parameters in novel ways, some measure PPN parameters that have not been strongly constrained to date, and some begin to enter the post-post Newtonian regime. What follows is a list of some of them. Detailed discussion of many of these projects can be found in these proceedings.

a) *Search for Gravitomagnetism*

According to general relativity, moving or rotating matter should produce a contribution to the gravitational field that is the analogue of the magnetic field of a moving charge or a magnetic dipole. The Relativity Gyroscope Experiment at Stanford University (GP-B) is in the advanced stage of developing a space mission to detect this phenomenon. A set of four superconducting, niobium-coated, spherical quartz gyroscopes will be flown in a low polar Earth orbit, and the precession of the gyroscopes relative to the distant stars will be measured. The predicted effect of gravitomagnetism is about 42 milliarcseconds per year, and the accuracy goal of the experiment is about 0.5 milliarcseconds per year. Another proposal to look for the effect of gravitomagnetism is to measure the relative precession of the line of nodes of a pair of LAGEOS satellites with supplementary inclination angles; the inclinations must be supplementary in order to cancel the dominant nodal precession caused by the Earth's Newtonian gravitational multipole moments. A third proposal envisages orbiting a superconducting, three-axis, gravity gradiometer around the Earth to measure directly the contribution of the gravitomagnetic field to the tidal

Table 2. Current Limits on PPN Parameters

PPN Parameter	Experiment	Value or limit	Remarks
γ	time delay	1.000 ± 0.002	Viking ranging
β	perihelion shift	0.99 ± 0.02	$J_2 \approx 10^{-7}$
$ \xi $	Earth tides	$<10^{-3}$	gravimeter data
$ \alpha_1 $	orbital preferred-frame effects	$<4 \times 10^{-4}$	combined solar system data
$ \alpha_2 $	Earth tides	$<4 \times 10^{-4}$	gravimeter data
	solar spin precession	$<4 \times 10^{-7}$	assumes alignment of solar equator and ecliptic are not coincidental
$ \alpha_3 $	perihelion shift	$<2 \times 10^{-7}$	
	acceleration of pulsars	$<2 \times 10^{-10}$	statistics of dP/dt for pulsars
$ 4\beta - \gamma - 3 - \frac{10}{3}\xi - \alpha_1 + \frac{2}{3}\alpha_2 - \frac{2}{3}\zeta_1 - \frac{1}{3}\zeta_2 $	Nordtvedt effect	<0.007	lunar laser ranging
$ \zeta_3 $	Newton's third law for the Moon	$<10^{-8}$	lunar acceleration

gravitational force. In these and other examples of gravitomagnetic effects, the PPN parameter combination measured is $\frac{1}{2}(1 + \gamma + \frac{1}{4}\alpha_1)$.

b) Geodetic Precession

The precession of a gyroscope, or of the axis of an orbit in the curved spacetime surrounding a distant body, depends on the PPN parameter combination $\frac{1}{3}(2\gamma + 1)$. The gyroscope experiment may measure this to better than 10^{-4} . The effect of this precession on the lunar orbit (conventionally called the de Sitter effect) has been seen at the 10 per cent level (Bertotti *et al.*, 1987).

c) Improved PPN Parameter Values

A number of advanced missions have been proposed in which spacecraft anchoring and improved tracking capabilities would lead to significant improvements in values of the PPN parameters, of J_2 of the Sun, and of \dot{G}/G . For example, a Mercury orbiter, in a 2-year experiment, with 3cm-range capability, could yield improvements in the perihelion shift to a part in 10^4 , in γ to 4×10^{-5} , in \dot{G}/G to 10^{-13} yr^{-1} , and in J_2 to a few parts in 10^8 . An Icarus lander could yield similar accuracies for the perihelion shift, γ and J_2 . A Phobos lander, with 1.5 years of data at 15 m-range uncertainty, could improve \dot{G}/G to $3 \times 10^{-12} \text{ yr}^{-1}$, and could lead to refined asteroid masses.

d) Probing Post-post-Newtonian Physics

It may be possible to begin to explore the next level of corrections to general relativity beyond the post-Newtonian limit, into the post-post-Newtonian regime. One proposal is POINTS, a precision optical interferometer in space with $\mu\text{arcsecond}$ accuracy. Such a device would improve the value of γ to the 10^{-6} level, and could detect the second-order term, which is of order 10 $\mu\text{arcseconds}$ at the limb. Such a measurement would be sensitive to a new "PPPN" parameter, which has not been measured heretofore. Here, the experimental effort to enter the PPPN arena will have to be accompanied by theoretical work to devise a simple, yet meaningful, PPPN extension of the PPN framework (see for example, Benacquista and Nordtvedt, 1988).

e) Tests of the Einstein Equivalence Principle

The possibility of performing an Eötvös experiment in space has been studied, raising the possibility of testing WEP to 10^{-18} . The gravitational redshift could be improved to a few parts in 10^6 in an advanced redshift experiment using a hydrogen maser clock in an Earth-orbiting satellite in an orbit of 0.5 eccentricity. A hydrogen maser on Starprobe would further improve the first-order redshift, and would be sensitive to second-order corrections (these corrections are still part of the post-Newtonian limit, and depend on γ and β). Other relativistic benefits of Starprobe would be an improvement in J_2 to 2×10^{-8} , in α_1 to 0.007; J_4 , and time variations in J_2 , might also be detectable.

f) Testing Unconstrained PPN Parameters

Improved limits on the parameter of the Nordtvedt effect (eq. 5), together with improved limits on such parameters as ξ , α_1 , and α_2 from other tests, could begin to constrain the conservation-law parameters ζ_1 and ζ_2 , which are only poorly constrained to date. Further constraints could be made possible by looking for small perturbative effects in Earth-satellite and Lunar orbits (Shahid-Saless and Ashby, 1988; Will, 1971).

IV. IS THE PPN FORMALISM THE LAST WORD?

The basis for this discussion has been the PPN formalism. It is important to keep in mind that this formalism is based on a particular set of assumptions, namely the validity of symmetric metric theories of gravity, and subjective criteria of simplicity of the forms assumed for the metric. Other assumptions could be made. The extension to post-post-Newtonian gravity by Benacquista and Nordtvedt (1988) does not assume a metric, rather it assumes a many-body Lagrangian for matter, and equations of motion for light, together with some criteria of symmetry. Other formalisms based on affine theories have been developed (Coley, 1983).

Occasionally alternative theories of gravity arise that do not fit the PPN framework and that achieve some measure of fame (or notoriety!) for one reason or another. A leading example of this is the Moffat non-symmetric gravitation theory (NGT). For the most recent summary, see Moffat and Woolgar, (1988). In this theory, the metric is not symmetric, therefore according to the standard terminology, it is not a metric theory. The theory contains a parameter l^2 which may have a microscopic interpretation as depending on some combination of baryon number, lepton number, fermion number or some other quantum number of the source of gravity. The theory is purported to agree with all experiments to date, although one constraint has been placed on it using the effect of dipole gravitational radiation in the "11 minute binary" 4U 1820-30 (Krisher, 1987).

The moral is that the PPN framework, albeit a very useful tool for analyzing experiment and theory, should not be used to shackle experimentalists to a given mode of investigation of the possibilities for experiments. Instead, theorists and experimentalists should work together to devise and understand meaningful new tests of the gravitational interaction.

ACKNOWLEDGEMENTS

This research was supported in part by the National Science Foundation under grant PHY 85-13953.

REFERENCES

- Bartlett, D. F., and van Buren, D. 1986, *Phys. Rev. Lett.*, **57**, 21.
Benacquista, M., and Nordtvedt, K. Jr. 1988, *Astrophys. J.*, **328**, 599.
Bender, P. 1988, private communication.
Bertotti, B., Ciufolini, I., and Bender P. L. 1987, *Phys. Rev. Lett.*, **58**, 1062.
Coley, A. A. 1983, *Phys. Rev. D*, **27**, 728; **28**, 1829; **28**, 1844.
Haugan, M. P., and Will, C. M. 1987, *Phys. Today*, **40** (5), 69.
Krisner, T. P. 1987, *Astrophys. J. Lett.*, **320**, L47.
Moffat, J. W., and Woolgar, E. 1988, *Phys. Rev. D*, **37**, 918.
Nordtvedt, K., Jr., 1987, *Astrophys. J.*, **320**, 871.
Robertson, D. S., and Carter, W. E. 1984, *Nature*, **310**.
Shahid-Saless, B., and Ashby, N. 1988, *Phys. Rev. D*, in press.
Taylor, J. H. 1987, In *General Relativity and Gravitation*, ed. M. A. H. MacCallum, Cambridge University Press, Cambridge, p. 209.
Will, C. M. 1971, *Astrophys. J.*, **165**, 409.
Will, C. M. 1981, *Theory and Experiment in Gravitational Physics*, Cambridge University Press, Cambridge.
Will, C. M. 1984, *Phys. Reports*, **113**, 345.
Will, C. M. 1987, In *300 Years of Gravitation*, ed. S. W. Hawking and W. Israel, Cambridge University Press, Cambridge, p. 80.

DISCUSSION

HELLINGS: Is it true that redshift, Eotvos, and local Lorentz invariance rule out all nonmetric theories of gravity? Could, say, some general affine theory not satisfy these limits?

WILL: The experiments in question place finite upper limits on the sizes of effects, that can be translated into constraints on the characteristics of certain classes of nonmetric theories (theories that fall within the Lightman-Lee THem framework, for example). Some theories within this class are ruled out, while others may fit within the constraints. Theories of another class, say, affine theories, may or may not fit the constraints. In other words, the experiments do not rigorously exclude all theories of a given type. On the other hand, the better the accuracy, the harder it is going to be for a given candidate non-metric theory to accommodate the constraints.

MATZNER: In the latest papers by Moffat (Moffat and Woolgar 1988), he forms a dimensionless ratio by dividing L^2 by m . Perihelion precession then depends on the difference of this ratio for the Sun and Mercury, for instance. This makes this ratio a very hard thing to observe in solar system tests. Could you comment on this?

WILL: This latest result appears to be (at last) a proper treatment of the equation of motion of bodies in Moffat's NGT. Unfortunately, many of the effects then depend on the difference of L^2/m between various bodies, so if L^2 is proportional to m , as would be true approximately if it were proportional to baryon number, then the effects will vanish. Although the limit obtained by Krisher (1987) from dipole gravitational radiation also depends on this difference, the limit may be interesting because the relativistic nature of the neutron star in 4U 1820-30 alters the value of L^2/m . I have recently noticed that the electromagnetic field equations in NGT violate EEP, so that it may be possible to place a very interesting limit on L^2/m for the Earth alone using the recent Galileo free-fall experiment with uranium and copper. Stay tuned for further details.

SHAPIRO: Has there been any independent corroborations of the solar-oscillation mode identifications made by Hill and his collaborators in their estimate of J_2 from their solar data? Would you agree that future solar oscillation experiments will solve the problem of J_2 ?

WILL: Unfortunately I am far from an expert in the subject of solar oscillations, so my discussion of the published results tends to be non-critical. Since we now have almost 10 years of data, and several published values for J_2 , I would urge one of the experts in the field to perform a critical review of the published results. This may go some way toward answering whether this technique can confidently pin down (or has already pinned down) J_2 , say to the level of 10^{-7} , or whether it could even reach the level of 10^{-8} , and thereby compete with proposals for a Mercury orbiter or Starprobe. Until then, it is difficult for neophytes like me to judge, say, Hill's mode identification or eigenfunction inversion technique against any one else's.